

# Fine tuning in low and large $\tan \beta$ regions in the $\text{cE}_6\text{SSM}$

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## Abstract

The Electroweak sector in  $E_6$  supersymmetric models is subject to a degree of fine tuning in the percent to permil level. This can be attributed to the experimental limits on both the mass of the  $Z'$  boson associated with the extra  $U(1)'$  symmetry in the model, as well as the masses of naturalness-related sparticles (which is a general source of tuning in supersymmetric models). The degree of tuning can be smaller than that in the minimal supersymmetric standard model with universal fundamental parameters (the constrained MSSM). We show this by quantifying the fine tuning in regions of the parameter space of the constrained exceptional supersymmetric standard model ( $\text{cE}_6\text{SSM}$ ) corresponding to values of  $\tan \beta$  below and above 10. It is found that, a Higgs mass  $m_h \sim 125$  GeV, a gluino mass  $m_{\tilde{g}} \sim 1.5$  TeV, and a  $Z'$  boson mass  $m_{Z'} \sim 3.8$  TeV correspond to fine tuning in the 0.2% (0.1%) level for  $\tan \beta = 30$  (5).

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# 1 Introduction

Naturalness, which can be understood as the requirement that observable quantities in a given model does not possess large and unexplained fine tuning (see [1]), has been a leading principal for developing theories beyond the Standard Model (SM). The quadratic sensitivity of the Higgs mass-squared parameter ( $m_H^2$ ) to the scale of new physics, be it the Planck scale at  $10^{19}$  GeV or a scale at which heavy masses may exist (e.g. the Grand Unification (GUT) scale  $M_{GUT} = 10^{16}$  GeV), has led the community to suggesting the existence of new physics at low scale near 1 TeV (see [2, 3]). This is due to the fact that in the absence of new physics at the low scale, the parameter  $m_H^2$ , which is proportional to the measured value of the vacuum expectation value (VEV) of the Higgs field ( $v = 246.22$  GeV), will need to be carefully fine tuned, order by order in perturbation theory, against the cutoff of the new scale (or any heavy mass), thereby destabilizing the Electroweak scale. For example, if the scale of such masses is the GUT scale ( $\sim 10^{16}$  GeV), then the degree of tuning is roughly 1 part in  $10^{32}$ .

Among the well-motivated and most studied theories beyond the standard model that might appear at the low scale are supersymmetric (SUSY) models, in which  $m_H^2$  is protected from large radiative corrections by the symmetry (for a pedagogical review see [4]). However, the LHC is pushing the scale of SUSY close to or above 1 TeV [5, 6], thereby placing the concept of Naturalness to the experimental test.

SUSY models differ in their predictions of observables, such as the mass of the Higgs and the Z bosons ( $m_Z$ ). While some models require large contributions from radiative corrections as in the MSSM, other models can accommodate a 126 GeV Higgs even at tree-level. The  $E_6$ SSM is such a model [7, 8] (introduced in Section 2). However, it is a general feature of SUSY models that, the more the SUSY scale is pushed up by experiments (i.e. more separation from the weak scale), the more fine tuning is required in order to correctly predict the measured values of observables. Additionally,  $E_6$  models like the  $E_6$ SSM have a new and distinct source of fine tuning which is the  $Z'$  boson [9] that was investigated in [10].

One of the implications of this tension between the Electroweak scale and the SUSY scale (also known as the little hierarchy problem) is that a given model or a specific point in the parameter space becomes less attractive from the point of view of Naturalness which is usually used as a criteria to favour models or points in the parameter space over others. It is important from model building point of view to learn the degree of fine tuning within a given model and whether or not it is possible to find regions in the parameter space that have low fine tuning and correct predictions for the values of observables.

This, then, can be directly related to testing the predictions of Naturalness at the LHC.

In this note, we probe regions in the parameter space of the cE<sub>6</sub>SSM with unexplored fine tuning and quantify it, thereby complementing the results in [10].

## 2 The exceptional supersymmetric standard model

The E<sub>6</sub>SSM is based on the exceptional Lie group E<sub>6</sub>, which contains SO(10) as a subgroup, of which  $SU(5)$  is a subgroup. It is possible then to decompose the fundamental representation of dimension 27 under  $SU(5) \times U(1)'$  as,

$$27 \rightarrow \underbrace{10_1 + \bar{5}_2}_{\text{Quarks \& Leptons}} + \underbrace{1_5}_{\text{Singlets}} + \underbrace{1_0}_{\text{R-H Nutrinos}} + \underbrace{\bar{5}_{-3} + 5_{-2}}_{\text{Higgs doublets \& Exotics}} \quad (1)$$

One requires three 27s to ensure an anomaly free model. Additionally, extra non-Higgs superfields denoted  $(H', \bar{H}')$  coming from other incomplete representations denoted  $(27', \bar{27}')$  are added in order to ensure gauge coupling unification. Thus, in the notation of the  $SU(5)$  group the complete matter content in the E<sub>6</sub>SSM is,

$$3 \underbrace{(\bar{5} + 10 + 1_0 + 1_5 + \bar{5} + 5)}_{27} + \underbrace{(H', \bar{H}')}_{27', \bar{27}'} \quad (2)$$

At the GUT scale, where the gauge couplings unify, the E<sub>6</sub> group breaks down to the group structure of the standard model with an additional  $U(1)'$  group,

$$SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)', \quad (3)$$

which survives to low energy scales ( $\sim 1$  TeV).

In order to prevent rapid proton decay, flavor-changing neutral currents, and allowing only third generation Higgs doublets ( $\hat{H}_{u,d}$ ) and SM singlet ( $\hat{S}_3$ ) to couple to matter superfields, a number of discrete symmetries is imposed, namely, an approximate  $Z_2^H$ , and either a  $Z_2^L$  or a  $Z_2^B$  which specifies two distinct models allowing the exotic matter to be either diquarks or leptoquarks.

The  $Z_2^H$  invariant superpotential reads,

$$\begin{aligned}
W_{\text{E}_6\text{SSM}} \approx & \lambda_i \hat{S}(\hat{H}_i^d \hat{H}_i^u) + \kappa_i \hat{S}(\hat{D}_i \hat{\bar{D}}_i) + f_{\alpha\beta} \hat{S}_\alpha(\hat{H}_d \hat{H}_\beta^u) + \tilde{f}_{\alpha\beta} \hat{S}_\alpha(\hat{H}_\beta^d \hat{H}_u) \\
& + \frac{1}{2} M_{ij} \hat{N}_i^c \hat{N}_j^c + \mu'(\hat{H}' \hat{\bar{H}}') + h_{4j}^E(\hat{H}_d \hat{H}') \hat{e}_j^c + h_{4j}^N(\hat{H}_u \hat{H}') \hat{N}_j^c \\
& + W_{\text{MSSM}}(\mu = 0),
\end{aligned} \tag{4}$$

where the indices  $\alpha, \beta = 1, 2$  and  $i = 1, 2, 3$  denote the generations.  $S$  is the SM singlet field,  $H_u$ , and  $H_d$  are the Higgs doublet fields corresponding to the up and down types. Exotic quarks and the additional non-Higgs fields are denoted by  $D$  and  $H'$  respectively.

In order to ensure that only third generation Higgs like fields get VEVs a certain hierarchy between the Yukawa couplings must exist. Defining  $\lambda \equiv \lambda_3$ , we impose

$\kappa_i, \lambda_i \gg f_{\alpha\beta}, \tilde{f}_{\alpha\beta}, h_{4j}^E, h_{4j}^N$ . Moreover, we do not impose any unification of the Yukawa couplings at the GUT scale.

Finally, investigations of the Higgs sector, sparticles mass spectrum, dark matter, gluino phenomenology, flavor physics, gauge coupling unification, and F-theory origins can be found in [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]

### 3 Fine tuning

In a given model, it is possible to quantify the fine tuning associated with observables by systematically studying their sensitivity to fractional variations in the GUT scale fundamental parameters. To capture and quantify this sensitivity, Ellis et. al. [27] proposed a measure that is widely used in the literature (e.g. [28, 29, 30, 31, 32, 33, 34, 35, 36]) and can be defined as,

$$\Delta_a = \left| \frac{d \ln M_Z}{d \ln a} \right|, \tag{5}$$

where  $M_Z$  is the mass of the Z boson, which can be expanded in terms of a set of fundamental parameters as,

$$\frac{M_Z^2}{2} \approx \sum_{i=1}^n F_i z_i a_i^2 \tag{6}$$

where,  $a$  denotes the fundamental parameters,  $z$  is the coefficient corresponding to each parameter, and is calculated numerically using the renormalisation group equations.  $F$  is some factor, possibly, involving  $\tan \beta$ .

Next, using Eq. 5 and following the process sketched in Fig. 1; a master formula for the fine tuning was derived and presented in [10] where the details of the semi-analytical procedure and code implementations are provided.

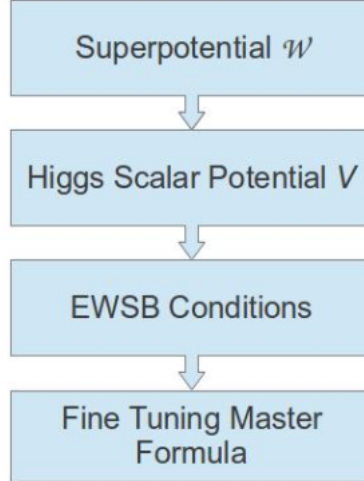


Figure 1: Process for deriving the master formula, where EWSB refers to the Electroweak symmetry breaking conditions obtained by minimizing the scalar Higgs potential.

In this study we scan the parameter space of regions where  $\tan \beta$  is either small  $\sim 5$  or large  $\sim 30$ . It is vital to note that changing  $\tan \beta$  affects the running of the renormalisation group equations which are used to expand low scale parameters in terms of high scale ones (see Eq. 6), hence allowing the quantifying of fine tuning in those distinct regions.

## 4 Results and discussion

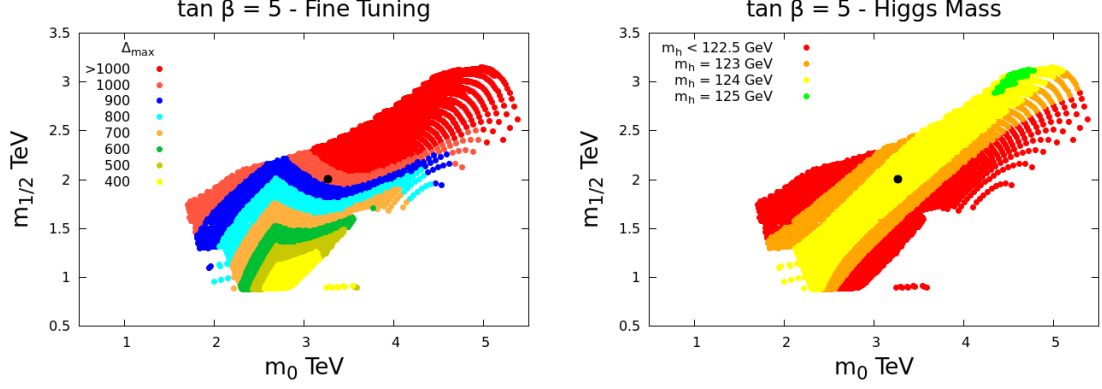


Figure 2: In the left panel, the fine tuning in the parameter space is shown in the  $m_0 - m_{1/2}$  plane, while the the right panel shows the values of  $m_h$ . All with a fixed value of  $M_{Z'} \approx 3.8$  TeV, and  $\tan \beta = 5$ .

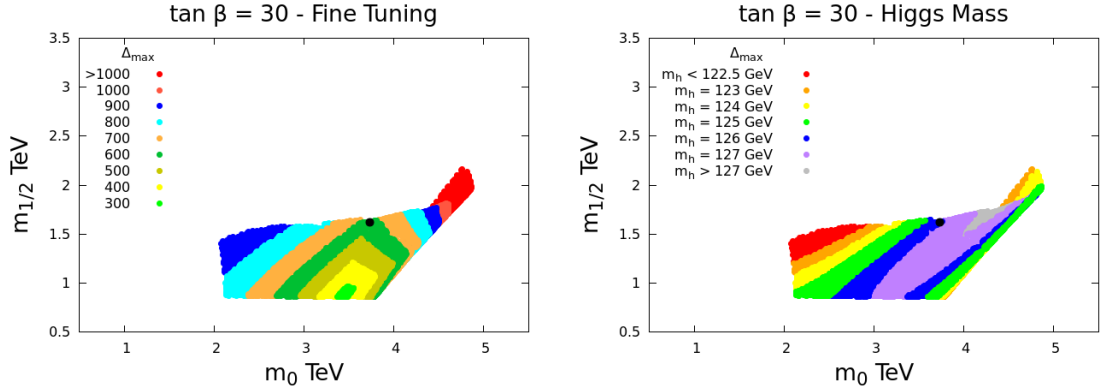


Figure 3: In the left panel, the fine tuning in the parameter space is shown in the  $m_0 - m_{1/2}$  plane, while the the right panel shows the values of  $m_h$ . All with a fixed value of  $M_{Z'} \approx 3.8$  TeV, and  $\tan \beta = 30$ .

Scanning over values of  $\lambda_3(\text{GUT}) \sim \{-3, 0\}$ ,  $\kappa_{1,2,3}(\text{GUT}) \sim \{0, 3\}$ , taking specific values of  $\tan \beta = 5$  and 30, while fixing  $s = 10$  TeV (i.e.  $M_{Z'} \approx 3.8$  TeV). The cuts we applied are rather conservative (see [10]) as we require a gluino mass  $m_{\tilde{g}} > 1.4$  TeV. The Higgs mass is required to be within the range  $123 < m_h < 127$  GeV.

From the right panel in Fig. 2, one can see that small values of  $\tan \beta$  can hardly produce a Higgs mass larger than 124 GeV. From our results we notice

that the gluino mass can be large ( $> 1.5$  TeV) in that region, however, fine tuning becomes larger as we approach lower values of  $\tan \beta$  (c.f. Fig. 3). The benchmark point (appears as a black dot in the Figures) for the  $\tan \beta = 5$  case corresponds to a Higgs mass of 124 GeV,  $m_{\tilde{g}} \sim 1.7$  TeV, and fine tuning  $\Delta \sim 1000$ , which is  $\sim 0.1\%$  tuning.

On the other hand, in regions where  $\tan \beta$  is large, as in Fig. 3, it is easy to find a Higgs mass above 125 GeV and fine tuning is slightly lowered. However, as one approaches larger and larger values it becomes somewhat difficult to find a gluino mass larger than 1.5 TeV. Therefore, moderate values of  $\tan \beta$  are favored in this model from phenomenological and naturalness standpoints. The benchmark point for  $\tan \beta = 30$  corresponds to  $m_h \sim 126.4$  GeV,  $m_{\tilde{g}} \sim 1.4$  TeV, and fine tuning in the  $\sim 0.2\%$  level ( $\Delta \sim 600$ ), which is slightly better than the previous case.

## 5 Conclusions

We have investigated regions of the parameter space of the cE<sub>6</sub>SSM where  $\tan \beta$  is as low as 5 and as high as 30. Moreover, we took into account the latest experimental limits on SUSY particles as well as the measured value of the Higgs boson. We find that, in general, fine tuning in the Electroweak sector lies in a level between  $0.2\% - 0.1\%$ . Small  $\tan \beta$  regions are characterized by Higgs mass between 123 and 125 GeV, and a gluino mass that can be larger than 1.5 TeV. The fine tuning is more severe in this region of the parameter space. On the other hand, large  $\tan \beta$  regions are associated with larger Higgs mass ranging from 123 to 127 GeV, but gluino mass that tend to be smaller than 1.5 TeV. However, the fine tuning is slightly less than that in the very low  $\tan \beta$  regime. We can then conclude that moderate values of  $\tan \beta$  are favoured by naturalness in the cE<sub>6</sub>SSM. Finally, in future studies, one can study the effects of including radiative corrections (e.g. the one-loop Coleman-Weinberg potential) into the definition of fine tuning.

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## References

- [1] G. 't Hooft, NATO Adv.Study Inst.Ser.B Phys. **59**, 135 (1980).
- [2] E. Gildener and S. Weinberg, Phys. Rev. D **13**, 3333 (1976).
- [3] L. Susskind, Phys.Rev. **D20**, 2619 (1979).
- [4] S. P. Martin, (1997), hep-ph/9709356.
- [5] CERN Report No. ATLAS-CONF-2013-053, 2013 (unpublished).
- [6] CERN Report No. ATLAS-CONF-2013-062, 2013 (unpublished).
- [7] S. King, S. Moretti, and R. Nevzorov, Phys.Rev. **D73**, 035009 (2006), hep-ph/0510419.
- [8] S. King, S. Moretti, and R. Nevzorov, Phys.Lett. **B634**, 278 (2006), hep-ph/0511256.
- [9] J. P. Hall and S. F. King, JHEP **1301**, 076 (2013), 1209.4657.
- [10] P. Athron, M. Binjonaid, and S. F. King, Phys.Rev. **D87**, 115023 (2013), 1302.5291.
- [11] P. Athron, S. King, D. Miller, S. Moretti, and R. Nevzorov, Phys.Rev. **D80**, 035009 (2009), 0904.2169.
- [12] P. Athron *et al.*, Phys.Lett. **B681**, 448 (2009), 0901.1192.
- [13] P. Athron *et al.*, Nucl.Phys.Proc.Suppl. **200-202**, 120 (2010).
- [14] P. Athron, S. King, D. Miller, S. Moretti, and R. Nevzorov, Phys.Rev. **D84**, 055006 (2011), 1102.4363.
- [15] P. Athron, D. Stockinger, and A. Voigt, Phys.Rev. **D86**, 095012 (2012), 1209.1470.
- [16] P. Athron, S. King, D. Miller, S. Moretti, and R. Nevzorov, Phys.Rev. **D86**, 095003 (2012), 1206.5028.
- [17] J. P. Hall and S. F. King, JHEP **0908**, 088 (2009), 0905.2696.
- [18] J. Hall *et al.*, Phys.Rev. **D83**, 075013 (2011), 1012.5114.
- [19] J. P. Hall and S. F. King, JHEP **1106**, 006 (2011), 1104.2259.



- [20] A. Belyaev, J. P. Hall, S. F. King, and P. Svantesson, Phys.Rev. **D87**, 035019 (2013), 1211.1962.
- [21] A. Belyaev, J. P. Hall, S. F. King, and P. Svantesson, Phys.Rev. **D86**, 031702 (2012), 1203.2495.
- [22] R. Howl and S. King, JHEP **0805**, 008 (2008), 0802.1909.
- [23] R. Howl and S. King, Phys.Lett. **B687**, 355 (2010), 0908.2067.
- [24] S. King, S. Moretti, and R. Nevzorov, Phys.Lett. **B650**, 57 (2007), hep-ph/0701064.
- [25] J. C. Callaghan and S. F. King, JHEP **1304**, 034 (2013), 1210.6913.
- [26] J. C. Callaghan, S. F. King, and G. K. Leontaris, (2013), 1307.4593.
- [27] J. R. Ellis, K. Enqvist, D. V. Nanopoulos, and F. Zwirner, Mod.Phys.Lett. **A1**, 57 (1986).
- [28] R. Barbieri and G. Giudice, Nucl.Phys. **B306**, 63 (1988).
- [29] P. H. Chankowski, J. R. Ellis, and S. Pokorski, Phys.Lett. **B423**, 327 (1998), hep-ph/9712234.
- [30] K. Agashe and M. Graesser, Nucl.Phys. **B507**, 3 (1997), hep-ph/9704206.
- [31] D. Wright, (1998), hep-ph/9801449.
- [32] G. L. Kane and S. King, Phys.Lett. **B451**, 113 (1999), hep-ph/9810374.
- [33] M. Bastero-Gil, G. L. Kane, and S. King, Phys.Lett. **B474**, 103 (2000), hep-ph/9910506.
- [34] Z. Kang, J. Li, and T. Li, JHEP **1211**, 024 (2012), 1201.5305.
- [35] S. Antusch, L. Calibbi, V. Maurer, M. Monaco, and M. Spinrath, JHEP **01**, 187 (2013), 1207.7236.
- [36] M. Perelstein and B. Shakya, (2012), 1208.0833.